Laboratory Tests for Characterizing Geomaterials

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Introduction

Engineering analysts are familiar with the uniaxial stress test used to characterize most metals, and used to calibrate the parameters associated with metal plasticity material models. However when the need arises to model geomaterials (concrete, rock, and soil), and some simple foams, the same analysts may be unfamiliar with the required suite of material characterization tests needed for calibrating the material model parameters in appropriate geomaterial constitutive models.

In this brief article a description of three common laboratory geomaterial tests are presented along with the corresponding material model parameters that are characterized by these tests. The tests covered are:

1. Hydrostatic compression
2. Triaxial compression/extension
3. Uniaxial strain

The material model parameters that can be calibrated to this data are used in the following LS-DYNA constitutive models:

- Soil and Foam (Material Type 5)
- Pseudo TENSOR (Material Type 16)
- Geologic Cap Model (Material Type 25)

Laboratory Test Specimens

The typical geomaterial laboratory test specimen is a right circular cylinder. For concrete the standard (United States) specimen has a 6 inch (152 mm) diameter and 12 inch (305 mm) height and is tested 28 days after the concrete is poured. More commonly, laboratory specimens have a 2 inch (51 mm) diameter and 4 inch (101 mm) height.

The cylinders are tested by applying axial and lateral loads (stresses) and recording the corresponding axial and lateral displacements (strains). The geometry of the cylinders, and applied loads, provides for an axisymmetric state of stress, and strain, in the cylinders that is typically denoted by the two principal stress components σ_1 and σ_3, where σ_1 is the stress applied in the axial direction and σ_3 is the lateral, or confining, stress applied to the cylindrical surface, see Figure 1. Note: the other principal stress, σ_2, by symmetry, is equal to the confining stress, i.e. $\sigma_2 = \sigma_3$.

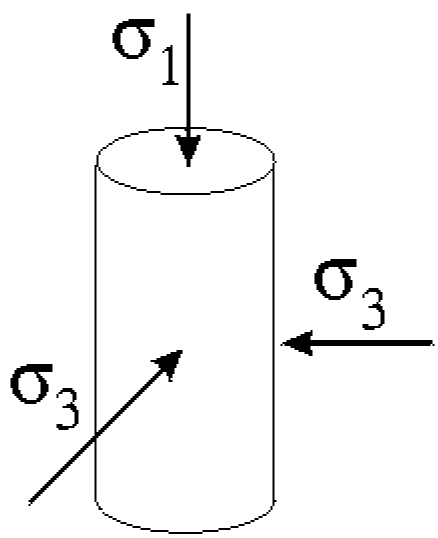

Figure 1 Typical geomaterial cylindrical laboratory specimen and axisymmetric loading.

Laboratory Tests

Figure 2 shows a schematic of the stress trajectories for five of the most common types of geomaterial characterization tests. Each of these five tests are briefly described in the sections that follow. After describing the tests, a summary of the geomaterial model parameters, for three LS-DYNA geomaterial constitutive models, is presented with an associated test to be used to calibrate each input parameter.

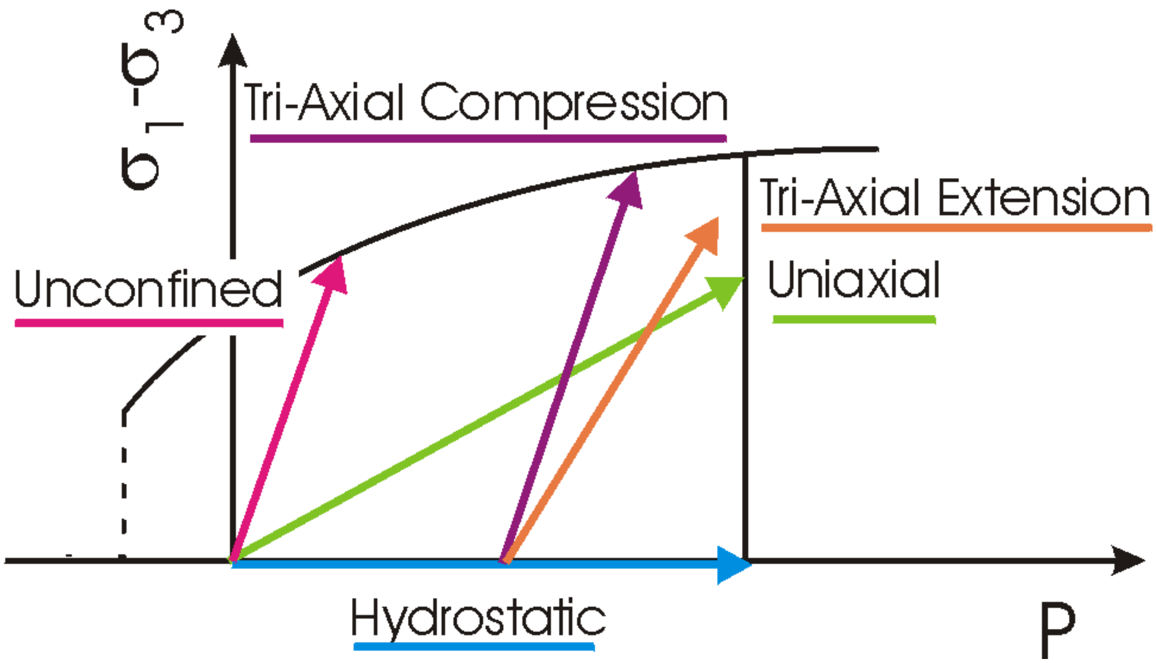

Figure 2 Stress trajectories for material characterization.

Hydrostatic Compression Tests

When the axial and lateral stresses are equal

$$\sigma_1 = \sigma_3 = \sigma$$

the specimen is in a state of hydrostatic compress (HSC) with a pressure

$$p = \sigma_{kk}/3 = \left(\sigma_1 + 2\sigma_3\right)/3 = \sigma$$

The corresponding measured axial and lateral strain components provide the volume strain

$$\varepsilon_v = \varepsilon_{kk} = \left(\varepsilon_1 + 2\varepsilon_3\right)$$

The corresponding pressure versus volume strain response describes the compaction behavior of the material as shown schematically in Figure 3. A typical geomaterial compaction response has three phases:

1. $p_0 < p < p_1$ is the initial elastic response. The elastic ***bulk modulus***, K, is the slope of this segment.
2. $p_1 < p < p_2$ is when the pores (voids) in the material are compressed,
3. $p > p_2$ removal of the voids results in a fully compacted material.

The indicated fourth phase is the unloaded from the fully compacted state. The slope of this segment defines the ***bulk unloading modulus***, K_{un} ,which is a user input for the Soil & Foam model (Material Type 5). Note the bulk unloading modulus should always be greater than the elastic modulus to prevent fictitious generation of energy during loading-unloading cycles.

It is important to note that, in general, LS-DYNA expects strain to be input as logarithmic strains. In the case of the volume strain, the measured (engineering) volume strain is related to the logarithmic volume strain by the simple relation

$$\ln\frac{V}{V_0} = \ln\left(1-\varepsilon_{kk}\right)$$

If the measured volume strains are great than about 10%, the conversion becomes important.

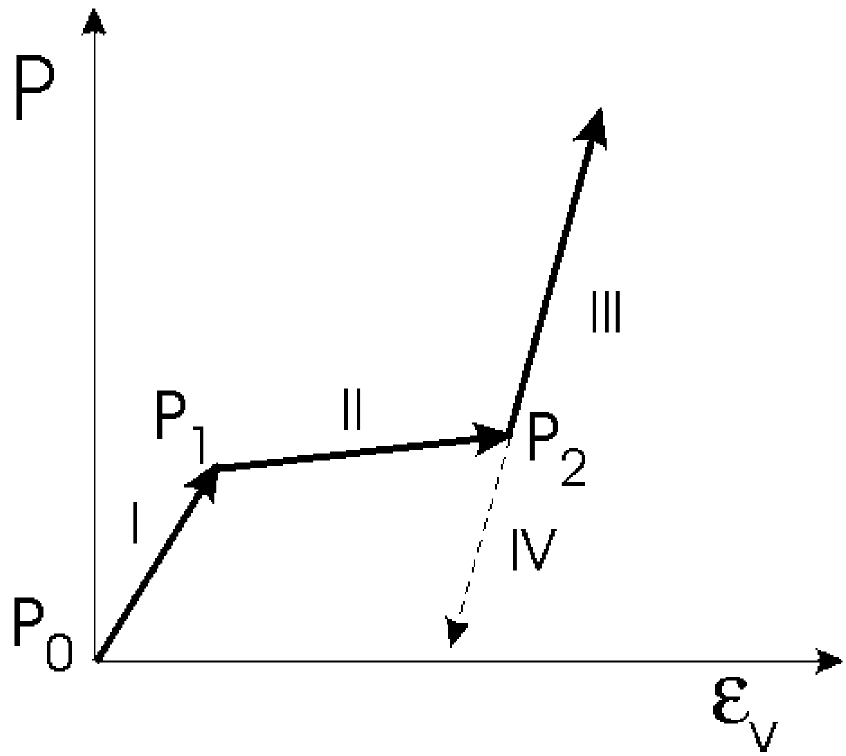

Figure 3 Schematic of pressure versus volume (compaction) response for a geomaterial.

Triaxial Tests

When the axial and lateral stresses are *not* equal, the test is generally referred to as a triaxial test. This is more a traditional name for the test rather than an accurate description of the test; there are so called 'true triaxial tests,' on cubical samples, where all three principal stress components are varied independently. Triaxial tests provide by far the most, and most important, data for characterizing the strength of geomaterials. Because the axial and lateral stresses are not equal, the cylindrical specimen is subjected to a shear stress, which is characterized by the stress difference between the axial and lateral stress

$$SD = \sigma_1 - \sigma_3$$

The stress difference, SD, is related to other useful stress invariant measures. The effective stress (von Mises stress), σ_e, is related to the second invariant of the deviatoric stress tensor via

$$\sigma_e = \sqrt{3J_2'}$$

and for a triaxial state of stress

$$\sigma_e = \sqrt{3J_2'} = \sigma_1 - \sigma_3$$

As with metals, it is the shear stress that is used to characterize the material's strength through the use of a constitutive model. However, unlike metals, the shear strength of geomaterials increases with increasing mean stress (pressure), as will be shown.

Unconfined Compression Test

A special case of the triaxial test is when the lateral (confining) stress is zero, i.e. $\sigma_3 = 0$, which is referred to an unconfined compression test (UCT). The corresponding value of the axial stress, when the specimen fails, is referred to as the unconfined compressive strength, and is usually denoted as σ_u. The unconfined compressive strength is an important measurement in characterizing geomaterials, as in a *sense* it provide the lowest estimate of the material's strength.

The importance of the unconfined compressive strength measurement in characterizing concrete is unprecedented in any other constitutive model, including metals. The simplest metal constitutive model requires two parameters: a yield strength and hardening modulus. However, concrete models exist that provided the elastic, shear strength, compaction, and failure response based solely on the unconfined compressive strength! The Pseudo-TENSOR model (Material Type 16) is one such model.

The initial elastic stress-strain response of an unconfined compression test can be used to calibrate Young's modulus and Poisson's ratio as it is easily shown from Hook's Law for uniaxial stress

$$\varepsilon_{axial} = \frac{\sigma_{axial}}{E}$$

$$\varepsilon_{lateral} = -\frac{\nu}{E}\sigma_{axial}$$

Triaxial Compression Tests

In the laboratory, a triaxial test is performed in two steps:

1. The specimen is loading in hydrostatic compression to a predetermined pressure,
2. Next either the lateral stress is held constant while the axial stress is increased, called a triaxial *compression* test, or
3. The axial stress is held constant and the lateral stress is increased, called a triaxial *extension* test; this test is discussed in the following subsection.

In a triaxial test, including unconfined, the mean stress is given by

$$p = \left(\sigma_1 + 2\sigma_3\right)/3$$

and for various values of the confining stress, σ_3, a data plot can be made of the stress difference versus mean stress when the laboratory specimen fails. Figure 4 shows a such a plot for a rock called Salem Limestone, which behaves much like a strong concrete, as reported by three laboratories. As can be seen in this figure, the limestone's shear strength, as characterized by the stress difference, increases with increasing mean stress (pressure). On this type of plot for a typical metal (von Mises model) the data would be aligned along a single horizontal line centered about the material's initial yield strength. A geomaterial's shear strength is characterized by fitting the constitutive model's shear failure surface to this collection of TXC data.

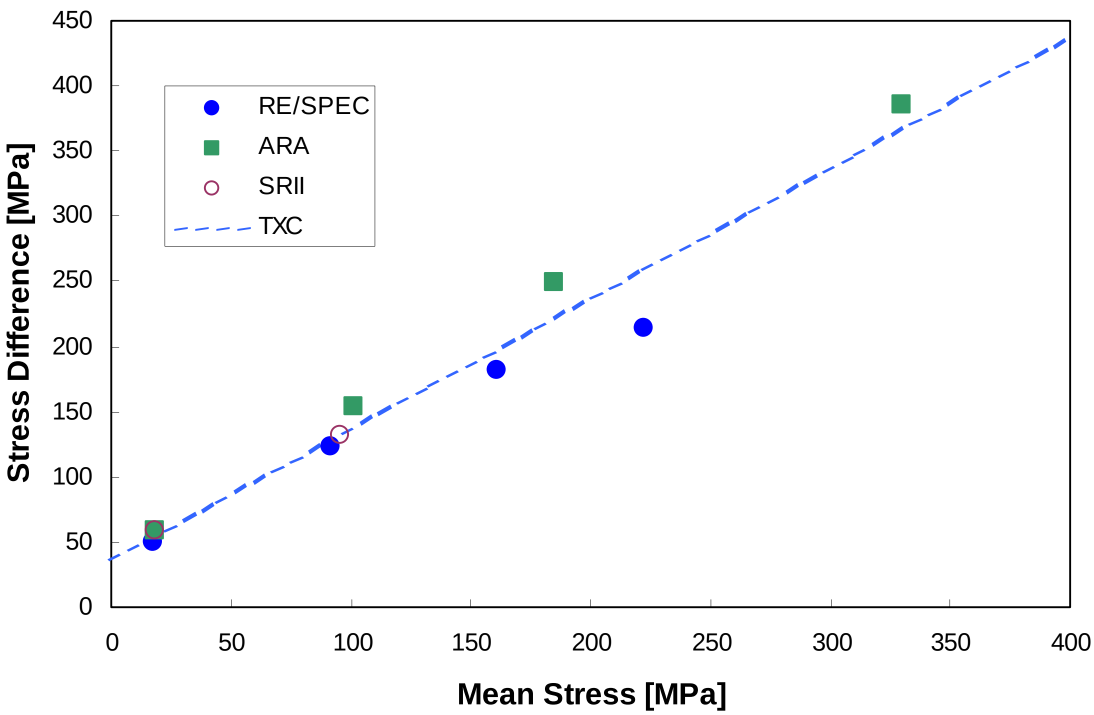

Figure 4 Triaxial compression laboratory data for Salem Limestone.

Triaxial Extension Tests

As mentioned above, a triaxial extension test differs from a triaxial compression test because for triaxial extension the lateral stress is increased while the axial stress is held constant; the reverse is true for triaxial compression. A simple view of this test, and a way to remember its name, is to image that as the lateral stress is increased the material wants to extend in the axial direction due to Poisson's effect.

The interesting phenomena exhibited by geomaterials in triaxial extension tests is that for a given mean stress the samples will fail at a lower stress difference. This is shown in Figure 5 where triaxial compression (TXC) and triaxial extension (TXE) data for Salem Limestone reported by ARA is plotted along with some straight line fits to the data.

The mode of specimen failure is also different. In triaxial extension the failure plane is normal to the minimum compressive stress (horizontal) and appears as an extensional failure, i.e. a tensile cleavage failure, whereas in triaxial compression the failure appears as a shear failure, i.e. typically on a 45° angle. In triaxial extension, the deviatoric axial strain is tensile which produces a 'tensile' type failure.

This phenomena is related to the relative ordering of the principal stresses, i.e.

$$\sigma_1 > \sigma_2 = \sigma_3 \quad \text{triaxial compression}$$
$$\sigma_1 < \sigma_2 = \sigma_3 \quad \text{triaxial extension}$$

and mathematically is usually characterized by J_3', the third invariant of the deviatoric stress tensor. Occasionally reference is made to three invariant models and this means that J_3', in addition to J_1 and J_2', is included in the model and hence the model can reproduce more realistic failure in triaxial extension.

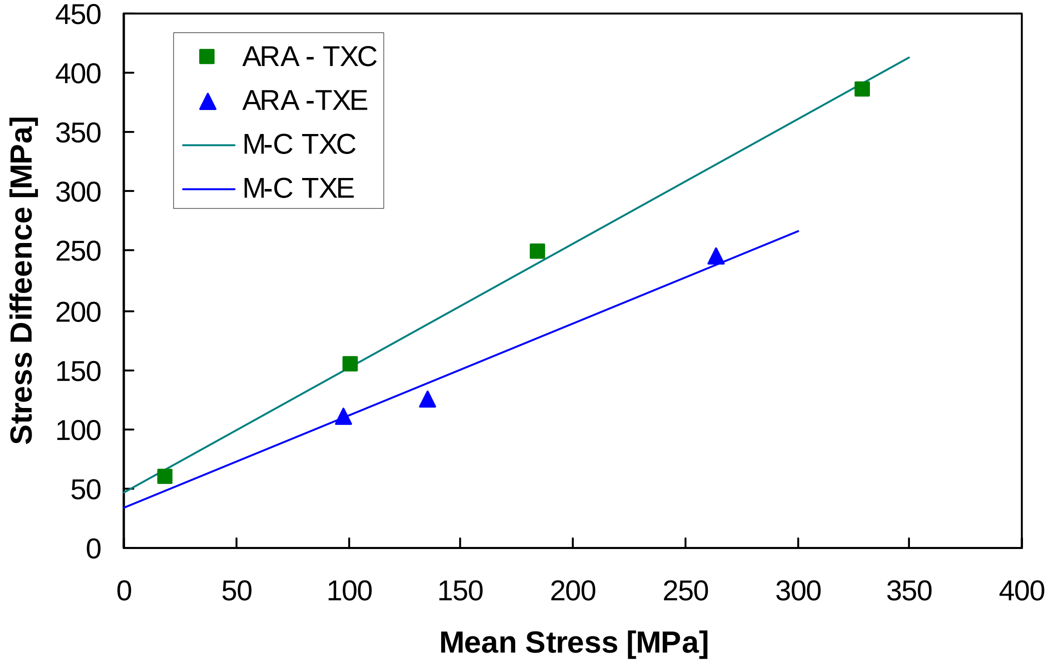

Figure 5 ARA tri-axial compression and extension data for Salem Limestone and the corresponding Mohr-Coulomb fits to the data.

Uniaxial Strain Tests

In a uniaxial strain test (UXE) the lateral confinement is continuously adjusted to maintain zero circumferential strain as the axial load is increased. Alternatively, the specimen is placed in a 'rigid' cylinder which prevents lateral displacement. This test is typically applied to soils, as a replacement for an unconfined compression test, which are inappropriate for most soils, i.e. they have almost no strength without some confinement. However, this is a very useful test for characterizing rocks and concrete, especially if a cap type model is used for the constitutive model. Figure 6 shows the stress difference versus mean stress data reported by three laboratories for Salem Limestone.

The initial slope of the uniaxial strain data can be used to calibrate the elastic Poisson's ratio, ν, or if the bulk modulus has been determined from the hydrostatic test data .shear modulus, G. It is straight forward to show, using Hook's Law, that for uniaxial strain:

$$SD = \frac{3(1-2\nu)}{1+\nu} p = \frac{2G}{K} p$$

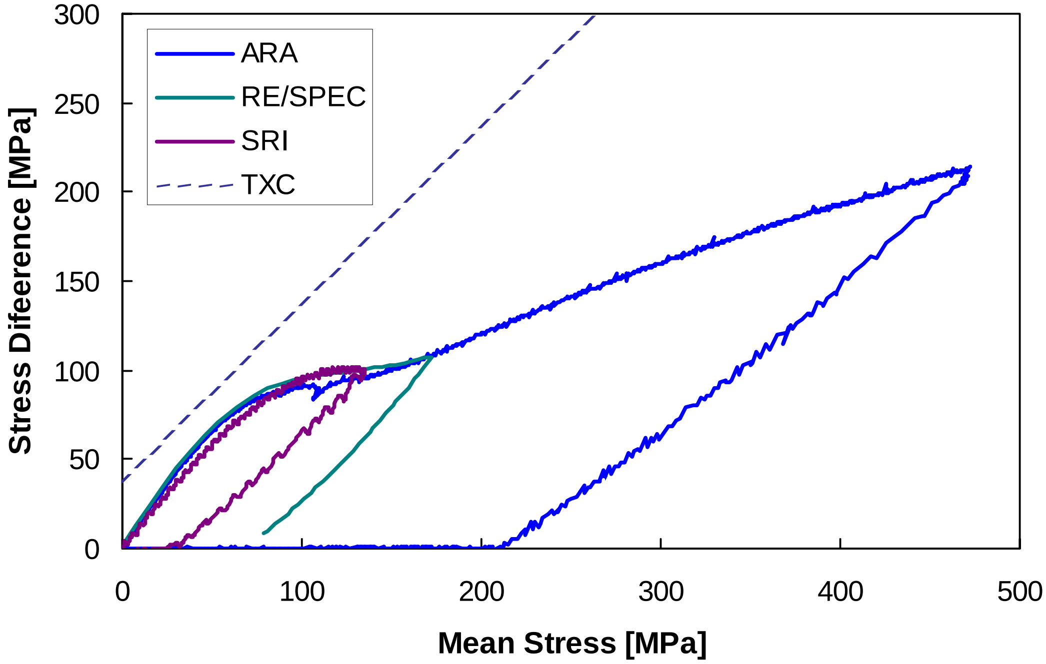

Figure 6 Stress trajectory from uniaxial strain test reported by three laboratories for Salem Limestone.

Material Parameter Calibration

Laboratory data from hydrostatic, triaxial and uniaxial strain tests can be used to calibrate all the material model parameters for three common LS-DYNA geomaterial models, as outlined in the following subsections.

Soil and Foam (Material Type 5)

The *Soil and Foam Model* is the most basic of the geomaterial models available in LS-DYNA. It is also the oldest and therefore has had a considerable amount of user experience, and feedback, and is quite robust. The model requires a minimum amount of input data, and hence material characterization. These facts make it the recommended model for preliminary analyses involving geomaterials, and for users new to modeling geomaterials.

Input	Test	Description
`G`	UCT or UXE	Elastic shear modulus
`BULK`	HSC	*Unloading* bulk modulus
`A0, A1, A2`	TXC	Shear failure surface parameters
`PC`	TXC	Mean stress intercept of shear failure surface
`VCR`		Use default VCR=0

REF		Use default REF=0
EPS1-10	HSC	Logarithmic volume strains
P1-10	HCS	Corresponding pressures (piecewise linear fit to HSC)

For this material model, the shear failure surface has the following functional form:

$$J_2' = a_0 + a_1 p + a_2 p^2$$

Pseudo-TENSOR (Material Type 16)

This model can be used in two modes:
- Response Mode I – Tabulated Stress Difference versus Pressure
- Response Mode II – Two Shear Strength Curves with Damage

Response Mode I is similar to the Soil & Foam model (Material Type 5). The shear failure surface is input in a tabular form and the compaction curve is also input in a tabular form via Equation of State 8 or 9.

Response Mode II has several further options, but perhaps the most useful is the *Concrete Model* option where the only required material characterization data is limited to the unconfined compressive strength $f_c' = \sigma_u$. The other significant feature of this model is the ability to include steel reinforcement (rebar) in a volume averaged sense, i.e. smeared rebar; this is only available in Response Mode II.

The Mode II model requires a minimum amount of input data, and hence material characterization, to use its basic features. The ability to specify only the unconfined compressive strength of the concrete of interest, and easily included smeared rebar, makes it the recommended model for preliminary analyses involving reinforced concrete. It is this Mode II input that is defined in the following table.

Input	**Test**	**Description**
G	UCT or UXE	Elastic shear modulus
PR	UCT or UXE	Poisson's ratio
SIGF	UCT	Unconfined compressive strength
A0		Use Mode II option A0=-1.0
A1, A2, A0F, A1F, B1		Use default = 0.0
PER, ER, PRR, SIGY,ETAN		Reinforcement properties
LCP, LCR		Load curve numbers for rate effects
X1-16		Use default = 0.0
YS1-16		Use default = 0.0

As mentioned in the LS-DYNA User Manual, when A0<0, the Equation of State number can be entered as zero and a tri-linear version of EOS 8 will be generated based on the specified unconfined compressive strength and Poisson's ratio.

If no laboratory data, other than the unconfined compressive strength f_c', is known for the concrete, the elastic modulus can be estimated from the formula provided by the American Concrete Institute: an estimate of Young's modulus for concrete, for a nominal weight density of 145 lbf/ft^3 , is given by the formula

$$E = 57,000\sqrt{f_c'} \text{ psi}$$

and typical values of Poisson's ratio for concrete range from 0.15 to 0.20.

Geological Cap (Material Type 25)

There are many variations on what is generically termed a *Cap Model*. These models have been developed from a long history, over 40 years, of two surface, i.e. shear failure and compaction, plasticity models. Models that use two independent non-intersecting surfaces, e.g. *Soil and Foam Model* and *Pseudo-TENSOR Model*, can be traced back to the ideas of Drucker et al. (1957). Most modern *Cap Models* that use two *intersecting* surfaces owe much to the seminal works of Sandler et al. (1976) and Sandler and Rubin (1979).

Input	Test	Description
BULK	HSC	Elastic bulk modulus
G	UCT or UXE	Elastic shear modulus
ALPHA, THETA, BETA, GAMMA	TXC	Shear failure surface parameters
R	UXE	Ellipticity of the cap
D, W, X0	HSC	Compaction surface parameters
C, N	UCT	Kinematic hardening parameters
FTYPE		=1 for soils and =2 for concrete and rocks
VEC		Vectorization flag
TOFF	TXC	Mean stress intercept of shear failure surface

This constitutive model's shear failure surface has the following functional form

$$\sqrt{J_2'} = F_e\left(J_1\right) = \alpha - \gamma \exp(-\beta J_1) + \theta J_1$$

The compaction response for this material model, and most cap type models, is specified via the relation

$$\varepsilon_v^p = W\left\{1 - \exp\left[-D\left(X - X_0\right)\right]\right\}$$

where ε_v^p is the plastic portion of the volume strain, X represents the first invariant of the stress tensor, i.e. $X = J_1 = 3p$, and W, D and X_0 are parameters obtained by calibrating the above expression to the hydrostatic compression data. While it is beyond the scope of the present article to describe the calibration procedure, it is fairly straight forward and is described in detail in the cap model section of the author's *Geomaterial Modeling with LS-DYNA* class notes.

The kinematic hardening parameters, see Figure 7, allows for an initial shear yield surface, located a distance N from the shear failure surface, to move (harden via the c parameter) toward the shear failure surface. Default, zero, values maybe used.

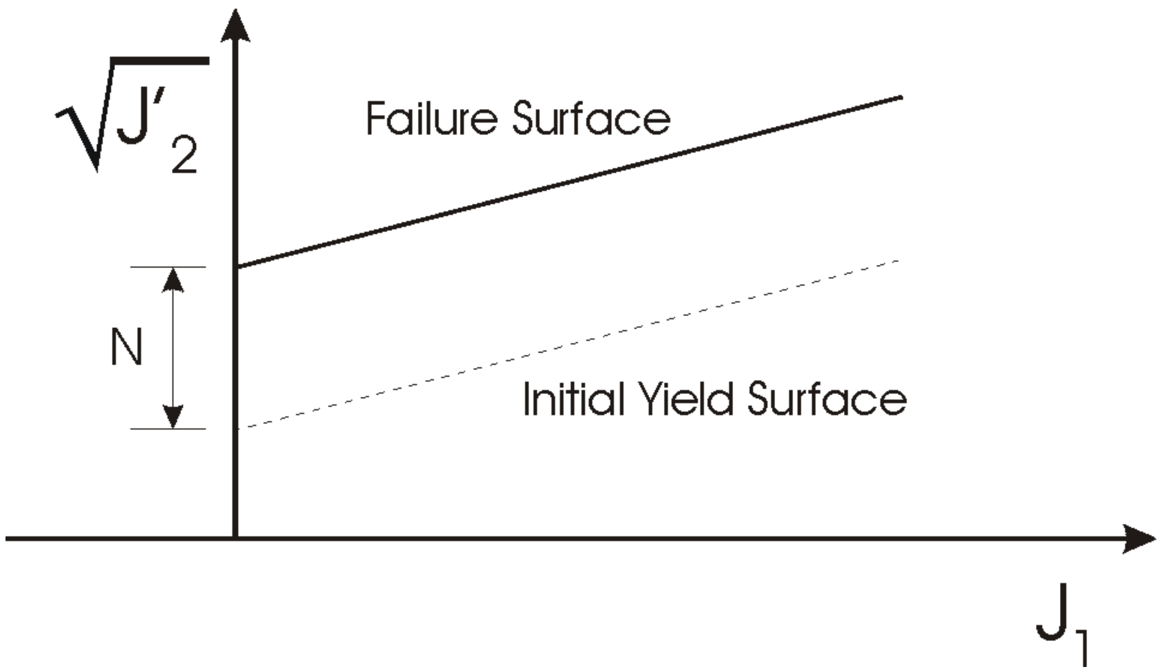

Figure 7 Initial location of the yield surface defined by the parameter N.

References

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Sandler, I.S., F.L. DiMaggio, and G.Y. Baladi, "Generalized Cap Model for Geological Materials," *Journal of the Geotechnical Engineering Division*, Volume 102, GT7, pages 683-699, July 1976.

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